Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students?

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Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students?

W Setyaningrum[1], A Mahmudi[2], and Murdanu[3]

[1]Mathematics Education Department, Yogyakarta State University, 1 Colombo Street, Yogyakarta, Indonesia

[1]wahyu_setyaningrum@uny.ac.id

Abstract. The aim of this paper is to describe pre-service mathematics teachers' knowledge about students. In this paper, knowledge about students refers to teachers' knowledge of mathematical concepts that are difficult for students, common misconceptions that frequently experienced by students, possible sources of students' errors, and methods to eliminate the difficulties and misconceptions. Twenty-four preservice mathematics teachers enrolled in micro-teaching course participated in the study. Interviews and written documents were employed to collect data. The data indicated that preservice teachers had problem in recognizing the source of students' misconceptions and error. Pre-service teachers also found difficulty in finding effective ways to minimize those misconceptions and error. Interview data revealed that pre-service teachers frequently failed to recognize students' misconception due to lack of conceptual knowledge of mathematics. Moreover, pre-service teachers had issues in generating effective teaching strategies as they had limited knowledge of teaching methods. The results suggest that mathematics pre-service teachers should have sufficient knowledge of both mathematics knowledge and pedagogic knowledge so that they can help their students in learning mathematics effectively.

1. Introduction

Teacher education program, in general, aimed to support preservice teachers develop professional knowledge for effective teaching. Therefore, the courses of the program cover content or subject matter and pedagogy knowledge. These two factors are integrated and could not be separated each other [1], as called pedagogical content knowledge (PCK) [2]. Pedagogical content knowledge consisted of two main aspects namely pedagogical knowledge and content knowledge. The first aspect relates to the knowledge about pedagogy such as how students learn, how to teach certain material or concepts, how to evaluate and assess students' understanding, etc. The later deals with the knowledge of the material in the case of mathematics teachers, it is about mastering the concepts in mathematics. Combining these two aspects, Shulman defined PCK as teachers' knowledge of representations, analogies, examples, and demonstrations to make a subject matter comprehensible to students [3]. In short, PCK is an integration of content and pedagogy knowledge concerning on understanding of how particular aspects of subject matter are structured, ordered, adapted and represented in teaching and learning processes.

PCK also related to knowledge of the students [4] that includes knowing students' needs, students' learning difficulties and misconceptions, and strategies on how students easily understand the subject [5,6]. Knowledge of the students also defined as knowledge about common difficulties and errors and

the ability to diagnose and eliminate them [7]. Many scholars agreed that one of the main aspects in the knowledge of students that is very important and need to be addressed by preservice mathematics teachers, is knowing the common mistakes students usually experienced and the reasons why they have such incorrect concepts or solutions [8,9]. So that the preservice teachers will be able to organize their pedagogical knowledge in order to make the teaching-learning process more effective.

There have been many research studies investigating the PCK of preservice mathematics teachers and knowledge about concepts of mathematics and how to teach those concepts to their students [3,10,11,12,13]. In the field of teachers' knowledge of the student, previous research revealed that teachers faced some problem in identifying source of incorrect answer or error done by their students [1].

In Indonesia, preservice teachers have to deal with many issues when they go to the classroom such as the differences of the character of the students, demographics background of the parent, school environment, and socio-cultural around the school. In addition, they also have to face the dynamic changes of the curriculum as a consequence they have to be aware of the changes in the material or the content of teaching. Therefore, PCK of the preservice teachers would be very important to cope with this situation.

Considering the important role of teachers' knowledge of the students and limited such research study conducted in Indonesian context, this study investigates the knowledge of pre-service mathematics teachers about their students. The teachers' knowledge of students in this study focuses on the extent to which preservice mathematics teachers know students' difficulties and possible and reasons behind the misconception that student might have on certain mathematical topic.

2. Method

This is a descriptive qualitative study focusing on exploring preservice mathematics teachers' understanding on how students' think in solving mathematics problems, what difficulties and error students' have, and why those difficulties and error happen. The participants were preservice students in third year of teacher education program who enrolled in micro teaching course. The reason of choosing these participants was that they had already took both compulsory content courses and pedagogical courses therefore they were considered to have enough knowledge on content and pedagogy.

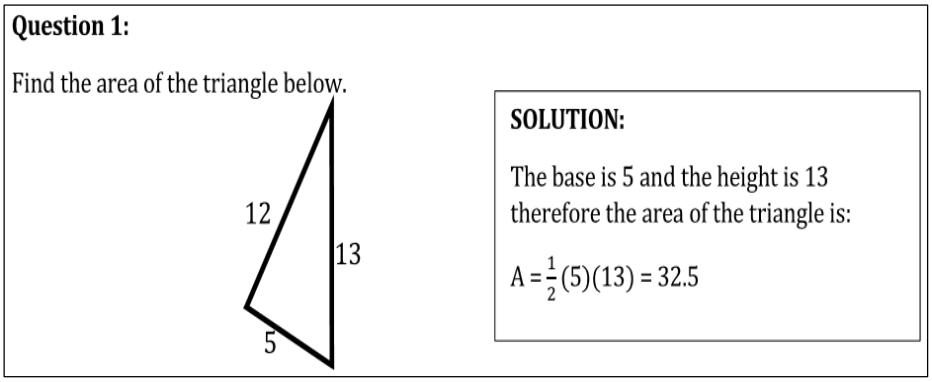


Figure 1. Question number 1.

The data collected from the questionnaire and interview. Twenty-four preservice mathematics students were asked to give comment on a set of questionnaires. The questionnaires contained two open ended questions consisted of mathematics problems about triangle and students' answers on those problems. The participants were asked to determine whether the students' answers were correct or incorrect and diagnose what was the possible explanation of the students' mistakes or misconceptions. The first question was about the area of the right triangle as shown in Fig. 1. This

question aimed to explore preservice teachers' knowledge of misconception on the height, base and the area of a right triangle (Fig.1). Therefore, the position of the right triangle is not usual and there is no sign for the right angle. The solution shown in the small box on the right-hand side is the given solution that is incorrect, it is chosen because it is one of the common mistake experience by students.

The second question constructed to investigate preservice teachers' understanding on the sides' properties of triangles. This question was about determining the length of the third side of a triangle if the length of other two side are given (Fig. 2). There were two incorrect solutions provided that were called "Solution student 1" and "Solution student 2". "Solution student 1" was the common error found at school meanwhile the "Solution student 2" was another error students' made.

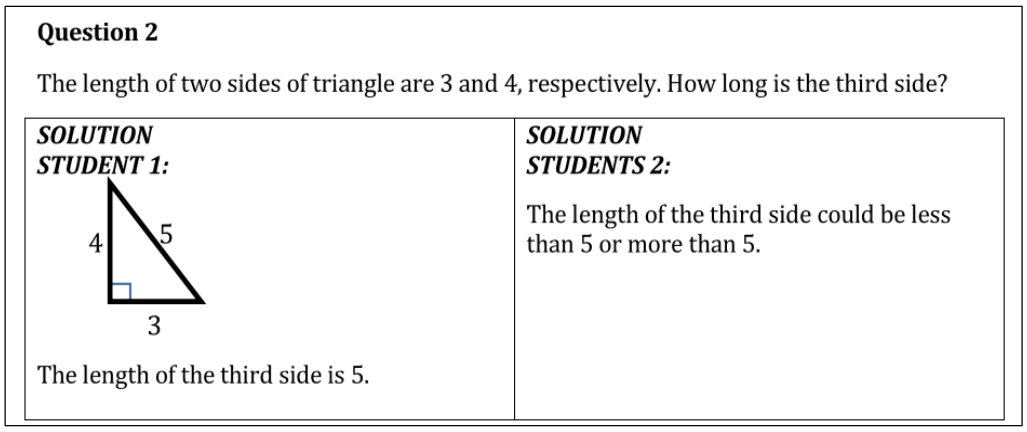


Figure 2. Question number 2.

Interview technique was employed to collect more information on the preservice teachers thought about their knowledge of students. The interview was focused on seeking explanation on the sources of students' mistake or misconceptions and its solution. Therefore, in the interview, the participants were asked to explain the reasons or sources of such mistakes and the method of teaching to overcome or to minimize this mistake based on their mathematical knowledge and pedagogical knowledge. Data gathered from questionnaire and interview were analyzed descriptively.

3. Result and Discussion

Data from the questionnaire and interview found that most of the preservice mathematics teachers had problems in identifying incorrect answers. The data of the first question shown at Fig. 3, revealed that more than half of the participants could identify the mistakes done by students and know the reasons why the students made that particular mistake. They claim that students made errors in identifying height and based of a right triangle incorrectly. The common source of mistakes identified by preservice teachers is that students confused in determining the height of a right triangle because the triangle is drawn in unusual position.

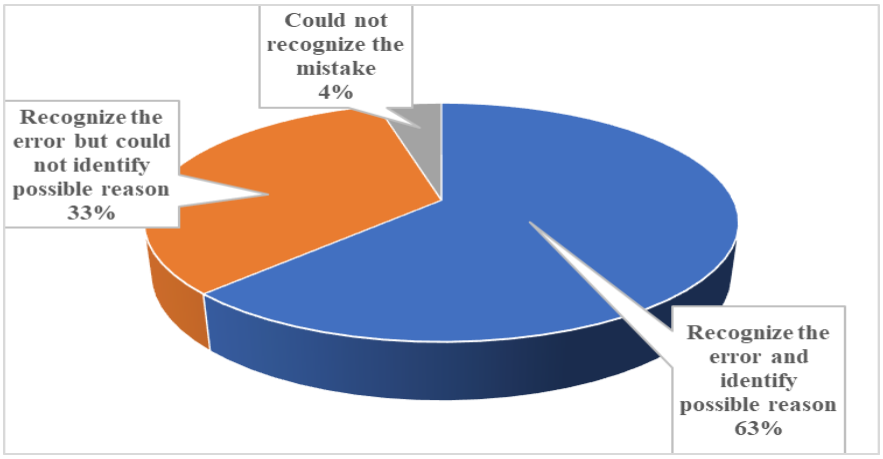


Figure 3. Preservice mathematics teachers' responses to question number 1.

The interview with those who could point out the misconception in the answer of the first question revealed that the process of teaching and learning at school might contribute to this such misconception. They argue that the default position of a right triangle taught at school represented in Fig. 4 in which the vertical line represents the height of a triangle and the horizontal line as the base of a triangle. Therefore, they suggest a certain method to eliminate this such mistake that is by representing triangles in many different positions while embedded the concepts of height and base of a right triangle (Fig. 5). They believe that exposing students to many different position of a right triangle could help them distinguish that the height of a triangle is not necessary represent in the vertical line and so for the base of a triangle.

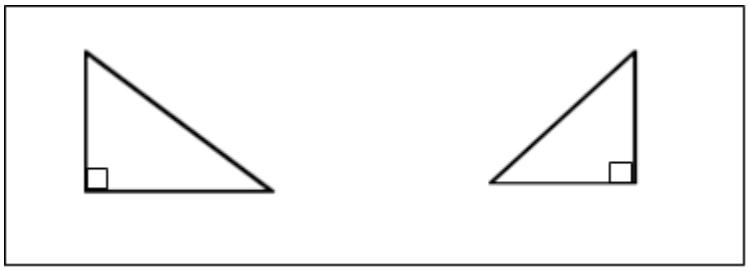


Figure 4. Common position of right triangles taught at school.

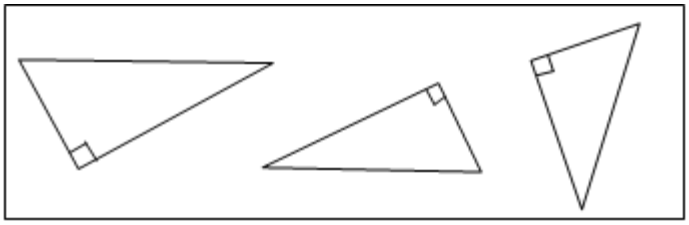
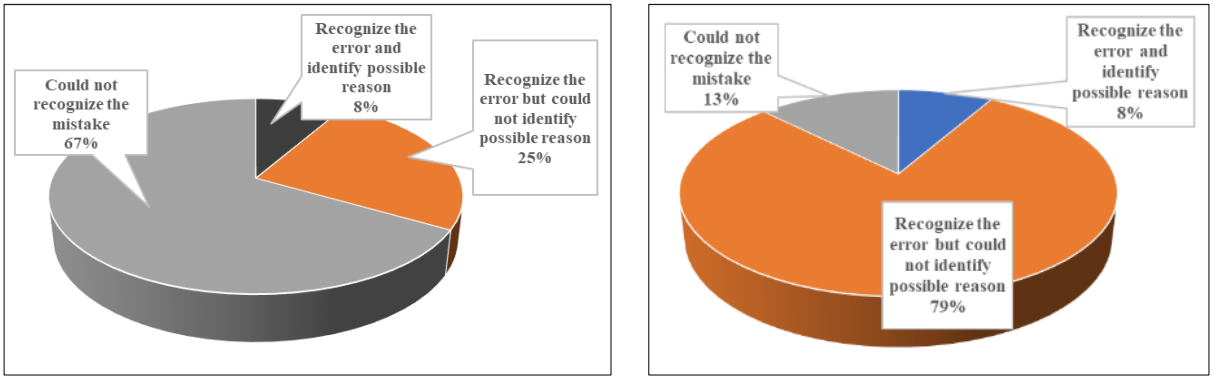


Figure 5. Other possible positions of right triangles should be introduced at school.

On the other hand, around a third of preservice teachers who participate in this study could recognize the error but failed to determine possible source of the error (Fig.3). They have no idea why students thought that the height is 13. The interview revealed that they more focus on the question rather than on the student's way of thinking, they argue that it is necessity to put a right sign on the right angle so that the students will recognize that the triangle is a right triangle. They did not mention the pedagogical as well as conceptual aspects on this issue.

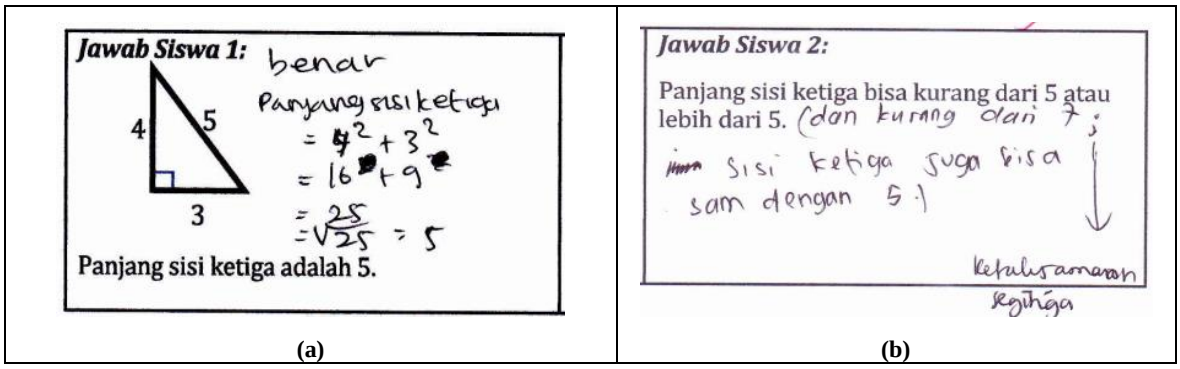
Interestingly, there are 4% of the participants who do not recognize the mistakes (Fig. 3). They believe that the given solution is correct. They just realize that the solution is incorrect during the interview. They thought that the height is 13 as they confuse with the picture in the question. Preservice teachers should not have this such misconception because this is a basic concept that teachers should acquire.

The data from question number 2 shows different trend to the data of question 1. The preservice teachers have problems in recognizing the incorrect answers as well as determining the source of mistakes and teaching method to deal with this such mistake. As can be seen in Fig. 6, most of the preservice mathematics teachers who participate in this study (67%) believed that "Solution student 1" was correct (Fig 6(a)). They argue that if the length of two sides of a triangle are 3 and 4 thus the third side should be 5 as they using the Pythagorean theorem Fig. 7(a). Further interview to this group of preservice teachers found that they only saw one possibility of the triangle that could be made that was a right triangle. They did not consider the other possibility such as acute triangle or obtuse triangle by using triangle inequality theorem.



(a) (b)

Figure 6. Percentage of the responses of question number 2.



(a) **(b)**

Figure 7. Preservice mathematics teachers' responses to question number 2

Meanwhile, the other 25% of the participants knew that "Solution student 1" is incorrect yet difficult to recognize the sources of the mistakes. They believe that the "Solution student 1" is only one of the possibility, but they either confused or forgot the concept related to this problem. There is only 8% of the participants who could identify the sources of mistakes. They argue that the student fixated to Pythagorean theorem and do not consider the other possibility by using the triangle inequality theorem (Fig. 7(b)). In regard to the effective way to minimize this mistake, they point out that teachers have to revisit the triangle inequality theorem while introducing the concepts of a right triangle and the properties of its sides.

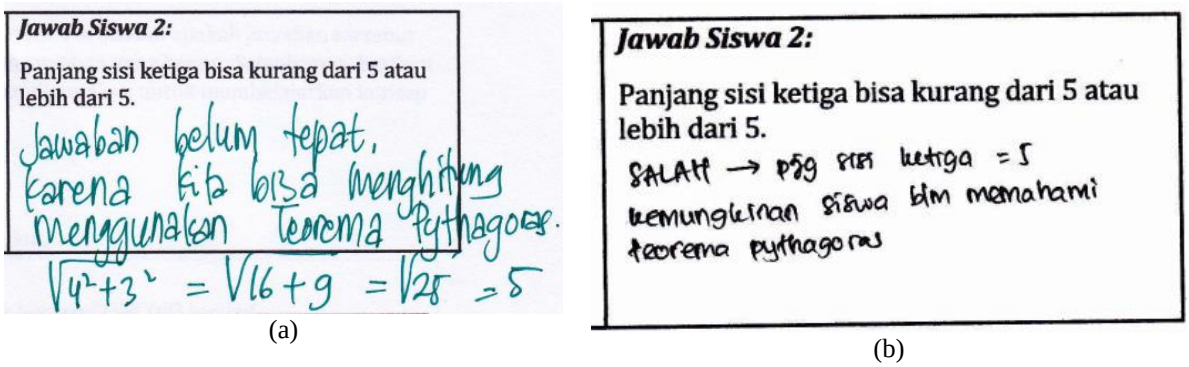


Figure 8. Preservice mathematics teachers' responses to Solution student 2 (*JawabSiswa* 2).

Responses for "Solution student 2" have similar pattern to the responses for "Solution student 1". Most of the participants (87%) believe that the solution is incorrect but only 8% who can recognize the reason of the mistakes (Fig. 6). Those who know the reason are same people as mention the triangle inequality theorem in the "Solution student 1". Meanwhile, the other provide inappropriate reason (Fig. 8) or do not point out any reason. Figure 8 shows that preservice mathematics teachers in this study consider "Solution student 2" as incorrect solution yet they mentioned inappropriate reason, they claim that students' should solve question no 2 using Pythagorean theorem so the correct answer should be 5. This implies that preservice teachers still have limited knowledge on the properties of sides in a triangle.

The rest 13% of the participant state that "Solution student 2" is the correct answer for question number 2. Some of them claim that the question does not refer to a right triangle therefore the length of the third side of the triangle could be less than or more than five (Fig. 9(a)). Meanwhile, some other preservice teachers in this group do not provide any explanation about their answers on "Solution student 2".

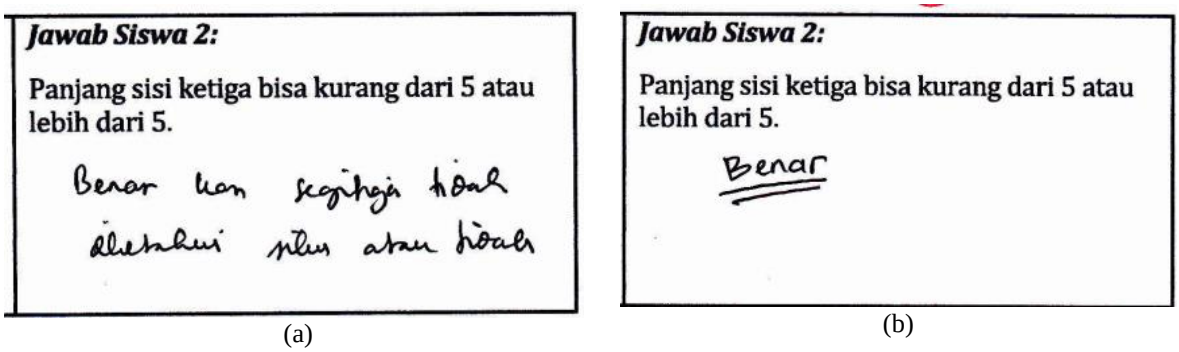


Figure 9. Preservice mathematics teachers failed to recognize errors.

Interview related to Question 2 suggest that most of the preservice mathematics teachers in this study still have limited knowledge on the concepts of triangle sides. Many of them forgot the triangle inequality theorem. In addition, some of the participant also failed to determine the source of mistakes experienced by students. The data in this study indicate that most of the preservice teachers in this study do not have a comprehensive knowledge about the concept of a right triangle, the properties of the sides of triangle, and the concepts related to them such as Pythagorean theorem and also the triangle inequality theorem.

Analysing the responses of the preservice mathematics teachers on the two questions above, it seems that they have problems in both identifying the students' mistakes and the source of students' errors or misconceptions. In addition, they frequently fail to recognize what conceptual knowledge

students lacking of as they have insufficient knowledge of mathematical contents as well as knowledge of the students. These findings support the earlier studies in this field that found preservice mathematics teachers still lacked knowledge of students' mathematical thinking [1,3,7,14,15]. The possible reason of lacking knowledge of students could be related to limited interaction with real students in the classroom, as they just about to start to teach at school.

Another significant finding of this study is that there is inadequate knowledge of the preservice teacher in generating effective ways to eliminate students' errors or misconceptions. This finding in line with the result previous studies that pointed out that there is a need for preservice teachers to be exposed to mathematics pedagogy during their study in teacher education program [16]. Considering the findings of this study, there are lots of works to do to improve both pedagogical and content knowledge of the preservice mathematics teachers.

4. Conclusion and Implication

This study revealed that the preservice mathematics teachers lacked both knowledge of students' mathematical thinking and knowledge of teaching. The possible reason of this limited knowledge is lacked of conceptual understanding of the material, in this study, preservice mathematics teachers failed to recognize the relationship among right triangle, Pythagorean theorem and the triangle inequality theorem. Another possibility reason is inadequate teaching experiences or interaction with children. Considering the findings of this study, the preservice teachers should be exposed to many examples of students' errors or misconception so that they can improve their knowledge of students' way of thinking as well as improve their knowledge on subject matter. Furthermore, the preservice teachers should be given opportunities to work with students to experience how to help students understand mathematics as well as determine how to eliminate students' errors and misconceptions so that they could improve their pedagogical knowledge. Microteaching and teaching practicum at school are also the way to help preservice teachers to develop their pedagogical content knowledge. This study portrays a picture of preservice mathematics teachers in regard to pedagogical and content knowledge yet it is just related to triangle concept and involved small number of preservice teachers. Therefore, a further study needs to include more mathematics concepts and more participants.

5. Acknowledgments

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